# MATHEMATICAL ANALYSIS OF THE PROCESS OF MOTION OF A VAPOR IN A QUASICLOSED VOLUME HAVING A TEMPERATURE GRADIENT

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We solve, in a one-dimensional approximation, the equation of gas dynamics for the steady-state flow of a vapor, taking into account condensation and revaporization. We compare the results of our theoretical calculations with those that we obtained by experimental means.

In [1], as a result of an experimental study of the spatial distribution of condensate thickness (of condensation speed) on the walls of a cylindrical chamber having a temperature gradient, a qualitative model was formulated for the mechanism of transport of matter during its vaporization in a quasiclosed volume.† It was shown there that the motion of the vapor in a quasiclosed volume in the interval of temperature conditions employed represents its gas-dynamic expansion with condensation and revaporization. Our concern in this paper is to give a quantitative description of this process.

The position of the critical section and the values of the vapor parameters at the initial gas-dynamic section‡ are given by [1]:

$$x_0 = \frac{T_0(1-\varkappa) + 1.3\,\dfrac{T_0^2}{T^*}}{\dfrac{dT_W}{dx}},$$

$$T_1 \simeq 0.7T_0,\ \rho_1 \simeq 0.33\ \rho_{\mathrm{sat}}(T_0),\ P_1 = \rho_1 \frac{RT_1}{\mu}, \tag{1}$$

$$u_1 = c_0 \sqrt{\gamma \frac{RT_1}{\mu}}, \quad G_1 = \rho_1 u_1 \frac{\pi d^2}{4},\ M_1 = \frac{u_1}{c_0} = 1.$$

Since in the part of the vapor motion from the initial gas-dynamic section to the critical section†† no condensation of particles on the chamber walls takes place and the vapor parameters do not vary, we can take an arbitrary section in the interval $x^* \le x \le x_0$ as the initial gas-dynamic section. Henceforth we take the initial gas-dynamic section as $x = x_0$.

For the motion of the vapor in the cylindrical chamber, a unique independent variable which defines the conditions of vapor motion is the gas outflow rate; it depends on only a single coordinate [1]

$$G_x = \rho_x u_x \frac{\pi d^2}{4}, \tag{2}$$

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† The method and the experimental results are given in detail in [1].

‡ Without losing the generality of the basic relations, we carry out our numerical solution for a diatomic vapor.

†† In our case $x^* < x_0$.

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and a connection between the seven vapor parameters in differential form may be established by means of the following relations [2, 3]:

$$(M_x^2-1)\frac{du_x}{u_x}=-\frac{dG_x}{G_x}, \tag{3a}$$

$$(M_x^2-1)\frac{dP_x}{P_x}=\gamma M_x^2\frac{dG_x}{G_x}, \tag{3b}$$

$$(M_x-1)^2\frac{d\rho_x}{\rho_x}=M_x^2\frac{dG_x}{G_x}, \tag{3c}$$

$$(M_x-1)^2\frac{dT}{T_x}=(\gamma-1)M_x^2\frac{dG_x}{G_x}, \tag{3d}$$

$$(M_x-1)^2\frac{dM_x^2}{M_x^2}=-2\left(1+\frac{\gamma-1}{2}M_x^2\right)\frac{dG_x}{G_x}. \tag{3e}$$

Solving the system of Eqs. (3a)-(3e) simultaneously, we can express all the vapor parameters in terms of the Mach number and its value at the critical section:

$$\frac{u_x}{v_{\text{crit}}}=M_x\sqrt{\frac{\frac{\gamma+1}{2}}{1+\frac{\gamma-1}{2}M_x^2}},\quad \frac{P_x}{P_{\text{crit}}}=\left(\frac{\frac{\gamma+1}{2}}{1+\frac{\gamma-1}{2}M_x^2}\right)^{\frac{\gamma}{\gamma-1}},$$

$$\frac{\rho_x}{\rho_{\text{crit}}}=\left(\frac{\frac{\gamma+1}{2}}{1+\frac{\gamma-1}{2}M_x^2}\right)^{\frac{1}{\gamma-1}},\quad \frac{T_x}{T_{\text{crit}}}=\frac{\frac{\gamma+1}{2}}{1+\frac{\gamma-1}{2}M_x^2}, \tag{4}$$

$$\frac{G_x}{G_{\text{crit}}}=M_x\left(\frac{\frac{\gamma+1}{2}}{1+\frac{\gamma-1}{2}M_x^2}\right)^{\frac{\gamma+1}{2(\gamma-1)}}.$$

Of all the variables in the system (4) only $G_x$ in a given case can be considered to be a known function of x. In order to find all the remaining parameters of the vapor as functions of the coordinate, it is necessary to express $M_x$ in terms of $G_x$ from the last equation of the system (4) and then to substitute into the remaining equations of the system. We express the quantity $G_x$ through the use of the mass flow conservation law (equation of continuity), taking into account condensation and revaporization of the vapor at the chamber walls.

We write the equation of continuity in the form

$$\frac{d}{dx}(\rho u)=\frac{4}{d}j_x, \tag{5}$$

where $j_x = 4G_x/\pi d^2$. When condensation and revaporization is present, $j = j_r - j_c$. In accordance with [1],

$$j_r(T_W)=\rho_{\text{sat}}(T_W)\beta\sqrt{\frac{RT_W}{2\pi\mu}}, \tag{6}$$

$$j_c(T_V)=\rho\,T_V\,\beta\sqrt{\frac{RT_V}{2\pi\mu}}. \tag{7}$$

Then

$$\frac{d}{dx}(\rho u)=\frac{4}{d}\beta\sqrt{\frac{R}{2\pi\mu}}\left[\rho_{\text{sat}}(T_W)\sqrt{T_W}-\rho(T_V)\sqrt{T_V}\right] \tag{8}$$

or, taking relation (2) into account,

$$\frac{dG_x}{G_x}=\left[\sqrt{\frac{RT_V}{2\pi\mu}}-\frac{\rho_{\text{н}}(T_W)}{\rho(T_V)}\sqrt{\frac{RT_W}{2\pi\mu}}\right]\frac{4dx}{4xd}. \tag{9}$$

We introduce the new coordinate $x' = x - x_0$.

Using Eqs. (3e), (4), and (9), we obtain

$$\frac{dM_{x'}}{dx'} = \sqrt{\frac{2}{\gamma\pi}} \cdot \frac{1 + \frac{\gamma - 1}{2} M_{x'}^2}{M_{x'}^2 - 1} \times \frac{2}{d} \left[ 1 - \frac{\rho_{sat}(T_W)}{\rho_1} \sqrt{\frac{T_W}{T_1}} \left( \frac{1 + \frac{\gamma - 1}{2} M_{x'}^2}{\frac{\gamma + 1}{2}} \right)^{\frac{1+\gamma}{2(\gamma-1)}} \right]. \qquad (10)$$

For a linear temperature variation along the chamber walls

$$T_W = T_{crit}(1 - kx'), \qquad (11)$$

where

$$k = \frac{T_{crit} - T_{ch}}{l T_{crit}} = \frac{dT_W}{dx} \cdot \frac{1}{T_{crit}}; \; l = L - x_0$$

In accordance with [4],

$$\rho_{sat}(T_W) = \frac{A}{T_W^{7/2}} \exp\left( -\frac{\eta T^*}{T_c} \right).$$

Since at the section $x = x_0$ we have the relation [1]:

$$\rho(T_{crit}) \sqrt{T_{crit}} = \rho_1 \sqrt{T_1},$$

then

$$A = \rho_1 \sqrt{T_1} T_{crit} \exp\left( \frac{\eta T_v}{T_{crit}} \right),$$

whence

$$\frac{\rho_{sat}(T_W)}{\rho_1} \sqrt{\frac{T_W}{T_1}} = \left( \frac{T_{crit}}{T_W} \right)^3 \exp\left( -\frac{\eta T^*}{T_W} + \frac{\eta T^*}{T_{crit}} \right). \qquad (12)$$

The change in the quantity $(T_{crit}/T_W)^3$ for a decrease in $T_W$ from $T_{crit}$ to $T^L_{ch}$ is small in comparison with the change in the exponential term, so that we can safely put $(T_{crit}/T_W)^3 \simeq 1$ in Eq. (12). In this case, taking Eq. (11) into account,

$$\frac{\rho_{sat}(T_W)}{\rho_1} \sqrt{\frac{T_W}{T_{crit}}} \simeq \exp\left[ -\frac{\eta T^* kx'}{T_{crit}(1 - kx')} \right]. \qquad (13)$$

Substituting relation (13) into Eq. (10) and putting it into dimensionless form, we obtain

$$\frac{dM}{d\xi} = \frac{2l}{d} \sqrt{\frac{2}{\gamma\pi}} \cdot \frac{1 + \frac{\gamma - 1}{2} M^2}{M^2 - 1} \times \left\{ 1 - \left( \frac{1 + \frac{\gamma - 1}{2} M^2}{\frac{\gamma + 1}{2}} \right)^{\frac{1+\gamma}{2(\gamma-1)}} \exp\left[ -\frac{\eta T^* kl\xi}{T_{crit}(1 - kl\xi)} \right] \right\}, \qquad (14)$$

where $\xi = x'/l$; $0 \le \xi \le 1$. For a diatomic gas, $\gamma = 7/5$; $\sqrt{2/\pi\gamma} = 0.67$.

We introduce the notation

$$a' = 0.67 \frac{2l}{d}; \; b = \frac{\eta T^* kl}{T_{crit}}, \; c = kl = \frac{T_{crit} - T_{ch}}{T_{crit}},$$

$$Q = \frac{1 + \frac{\gamma - 1}{2} M^2}{\frac{\gamma + 1}{2}} = \frac{1 + 0.2 M^2}{1.2}, \; a = \frac{\gamma + 1}{2} a' = 1.2a'. \qquad (15)$$

Finally,

$$\frac{dM}{d\xi} = \frac{aQ}{M^2 - 1} \left[ 1 - Q^3 \exp\left( -\frac{b\xi}{1 - c\xi} \right) \right]. \qquad (16)$$

TABLE 1. Values of the Coefficients $m_1$, $m_2$, $m_3$

| Material | d, cm | L, cm | $l$, cm | $T_0$, °C | $dT_W/dx$, deg·cm$^{-1}$ | $m_1$ | $m_2$ | $m_3$ |
|---|---|---|---|---|---|---|---|---|
|  |  |  | 6,4 | 550 |  | 2.28 | —4.56 | 14.2 |
|  |  |  | 6,2 | 600 |  | 2.12 | —3,3 | 12,6 |
|  |  |  | 6.0 | 650 | 30 | 1.91 | —2,65 | 8,87 |
|  |  | 8,0 | 5.8 | 700 |  | 1.75 | —2.0 | 6,4 |
|  | 5,2 |  | 5.3 |  | 20 | 1,39 | —1 | 3,13 |
|  |  |  | 6.2 | 600 | 30 | 2.12 | —3,3 | 12,6 |
| CdS |  |  | 6,7 |  | 40 | 2.65 | —5.75 | 15,5 |
|  |  | 8.0 | 6.2 |  |  | 2.12 | —3.3 | 12.6 |
|  |  | 6,0 | 4,2 | 600 | 30 | 1,65 | —1,38 | 3,92 |
|  |  | 4.0 | 2,2 |  |  | 0,73 | —0,4 | 0,61 |
|  | 5,2 |  |  |  |  | 2.12 | —3,3 | 12,6 |
|  | 8,0 | 8,0 | 6,2 | 600 | 30 | 1,61 | —2,26 | 5,25 |
| CdS | 5.2 |  | 6.2 |  |  | 2,12 | —3,3 | 12.6 |
| CdSe |  | 8,0 | 6,05 | 600 | 30 | 2.03 | —2,9 | 10.9 |
| CdTe |  |  | 5,85 |  |  | 1.9 | —2,4 | 8.6 |

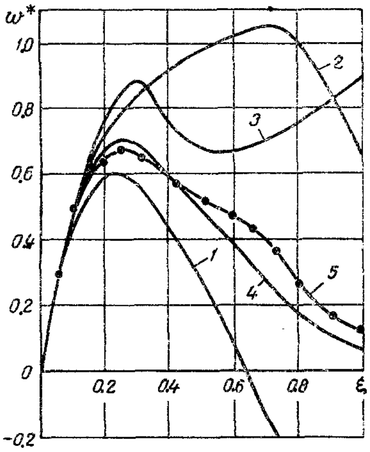

Fig. 1. Comparison of experimental results (5) with values of $w^* = w^*(\xi)$, calculated from various approximate formulas (1-4): 1) from Eq. (22), using two terms of the expansion (21); 2) from Eq. (22), using three terms of the expansion (21); 3) from Eq. (23), using three terms of the expansion (18); 4) from Eq. (23), using four terms of the expansion (18).

Equation (16) cannot be solved in general form except numerically; however, taking realistic values of the parameters appearing in Eq. (16), we can solve Eq. (16) approximately with sufficient accuracy in two extreme cases (close to the bottom or close to the top of the chamber) and then combine the two solutions. For small $\xi$ $(0 < \xi \leqslant 0.5)$, we can expand the solution of Eq. (16) in a Maclaurin series:

$$M(\xi) = M|_{\xi=0} + M'|_{\xi=0}\,\xi + \frac{1}{2!}M''|_{\xi=0}\,\xi^2 + \frac{1}{3!}M'''|_{\xi=0}\,\xi^3 + \ldots \qquad (17)$$

From Eqs. (1) we have $M|_{\xi=0} = 1$.

Let

$$M'|_{\xi=0} = m_1;\quad M''|_{\xi=0} = m_2;\quad M'''|_{\xi=0} = m_3.$$

Then

$$M(\xi) = 1 + m_1\xi + \frac{1}{2}m_2\xi^2 + \frac{1}{6}m_3\xi^3 + \ldots \qquad (18)$$

The problem now reduces to finding the coefficients $m_1$, $m_2$, $m_3$ (see the Appendix).

In Table 1 we present values of the coefficients $m_1$, $m_2$, $m_3$ for certain cases which reflect the influence of the various factors (the temperature conditions, chamber geometry, nature of the material) on the process of mass transfer and vapor condensation in a quasiclosed volume. Substituting the values of $M(\xi)$ into Eq. (4), we can determine the other parameters for the vapor moving in the chamber. The speed of the resulting condensation of the vapor on the walls of the chamber is $w = -dG/d\xi$ (the minus sign takes account of a decrease in the amount of vapor during condensation).

From Eqs. (4)

$$G(\xi) = G_1 M(\xi)\left[\frac{1.2}{1 + 0.2M^2(\xi)}\right]^3,$$

then

$$w = -\frac{dG}{d\xi} = -\frac{dG}{dM}\cdot\frac{dM}{d\xi}, \qquad (19)$$

$$-\frac{dG}{dM} = -\left(\frac{1.2}{1 + 0.2M^2}\right) - 3M\left(\frac{1.2}{1 + 0.2M^2}\right)\frac{(-1.2)\cdot 0{,}4M}{(1 + 0.2M^2)^2} = \frac{M^2 - 1}{1.2}\left(\frac{1.2}{1 + 0.2M^2}\right)^4, \qquad (20)$$

$$\frac{dM}{d\xi} = m_1 + m_2\xi + \frac{1}{2}m_3\xi^2 + \ldots \qquad (21)$$

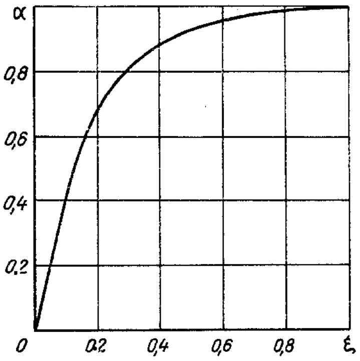

Fig. 2

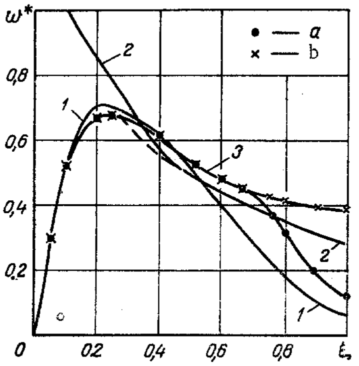

Fig. 3

Fig. 2. Variation of the condensation coefficient of CdS vapors along the length of the chamber ($T_0$ = 550°C, $dT_W/dx$ + 30 deg · $cm^{-1}$).

Fig. 3. Comparison of experimental results (3a, 3b) and calculated values (1, 2) of $w^* = w^*(\xi)$ for CdS vapors ($T_0$ = 550°C, $dT_W/dx$ = 30 deg · $cm^{-1}$): 1) taking account of revaporization; 2) neglecting revaporization; 3a, 3b) for plates placed parallel and perpendicular to the flow, respectively.

Substituting (20) and (21) into Eq. (19), we obtain, for the relative speed of condensation

$$w^* = w/w_1,$$

where

$$w_1 = \frac{u_1\rho_1 d}{2\rho_m l},$$

$$w^* = \frac{M^2-1}{1.2}\left(\frac{1.2}{1+(0.2)M^2}\right)^4\left(m_1 + m_2\xi + \frac{1}{2}m_3\xi^2 + \dots\right). \tag{22}$$

More accurate values of $w^*(\xi)$ may be obtained by a substitution for $dM/d\xi$ from Eq. (16):

$$w^*(\xi) = a'\left[\left(\frac{1,2}{1+0,2M^2}\right)^3 - \exp\left(-\frac{b\xi}{1-c\xi}\right)\right]. \tag{23}$$

In Fig. 1 experimental results are compared with various approximate calculated condensation speeds for $T_0$ = 550°C. The best agreement with experiment is furnished when $w^*(\xi)$ is calculated from Eq. (23), using four terms of the Maclaurin series (18) for the expansion of the function $M(\xi)$.

For values of $\xi \leq 0.4$ the experimental and theoretical results agree almost exactly. As $\xi$ increases, the accuracy of the solution of Eq. (16) in the form of the expansion (17) decreases (as $\xi$ increases the formula gives smaller values of $w^*$); this may be explained by the relatively slow convergence of the Maclaurin series (18).

We estimate now the contribution of the revaporization of the particles in the mass transfer of the vapor on the chamber walls as a function of the magnitude of the condensation coefficient $\alpha = (w_c - w_r)/w_c$. It is not difficult to show that in our case

$$\alpha = 1 - \left(\frac{1+0.2M^2}{1.2}\right)^3 \exp\left(-\frac{b\xi}{1-c\xi}\right). \tag{24}$$

It is obvious that when $\xi = 0$, $w_c = w_r$ and $\alpha = 0$. In the absence of revaporization, $w_r = 0$ and $\alpha = 1$.

We see from Fig. 2 that as $\xi$ increases, the revaporization contribution diminishes rapidly, and for $\xi \simeq 0.5$-$0.8$ the condensation coefficient is close to one. Therefore on this part of the chamber the motion of the vapor can be fairly accurately described by the function $M(\xi)$, which does not take into account revaporization of the vapor from the walls of the chamber, i.e., we assume that an arbitrary vapor molecule, impacting on the wall, condenses on it. In this case, Eq. (16) assumes the form

$$\frac{(M^2-1)\,dM}{1+\frac{\gamma-1}{2}M^2} = \sqrt{\frac{2}{\pi\gamma}}\cdot\frac{2l}{d}\cdot d\,\xi. \tag{25}$$

The left side of the resulting equation can be simplified by expanding the left member so as to involve M in a proper fraction. As a result the equation assumes the form

$$\frac{2}{\gamma-1}dM - \frac{\gamma+1}{\gamma-1}\cdot\frac{dM}{1+\frac{\gamma-1}{2}M^2} = a'd\xi.$$

After integrating, we obtain

$$\frac{2}{\gamma-1}M - \frac{\gamma+1}{\gamma-1}\sqrt{\frac{2}{\gamma-1}}\,\mathrm{arc\,tg}\sqrt{\frac{\gamma-1}{2}}M = \sqrt{\frac{2}{\pi\gamma}}\cdot\frac{l}{2d}\xi + C.$$

We determine the constant C from the condition $M|_{\xi=0} = 1$,

$$C = \frac{2}{\gamma-1} - \frac{\gamma+1}{\gamma-1}\,\mathrm{arctg}\sqrt{\frac{\gamma-1}{2}};$$

then

$$a'\xi = \frac{2M}{\gamma-1} - \frac{\gamma+1}{\gamma-1}\sqrt{\frac{2}{\gamma-1}}\,\mathrm{arctg}\sqrt{\frac{\gamma-1}{2}}M - \frac{2}{\gamma-1} + \frac{\gamma+1}{\gamma-1}\sqrt{\frac{2}{\gamma-1}}\,\mathrm{arctg}\sqrt{\frac{\gamma-1}{2}} \tag{26}$$

For $\gamma = 7/5$ we obtain the following relation for calculating M:

$$a'\xi \simeq 5M - 13.4\,\mathrm{arctg}\,(0.447M) + 0.65. \tag{27}$$

The relation (27) involves M implicitly; however, various values of M may be assigned and corresponding values of $\xi$ calculated; a table or graph of $M = M(\xi)$ can then be constructed.

The expression for the relative speed of condensation in the absence of revaporization has the form

$$w^* = a'\left(\frac{1.2}{1+0.2M^2}\right)^3. \tag{28}$$

In Fig. 3 we present both experimental and calculated values of the function $w^* = w^*(\xi)$ for $T_0 = 550°C$; the calculated values being determined from Eqs. (22) and (28). The experimental vapor condensation speeds of condensation on the chamber walls were also reduced to dimensionless form by dividing by $w_1$.

From Fig. 3 we see that in the interval $0 < \xi \leqslant 0.5$, curve 1 (in which revaporization is taken into account) gives a much closer agreement with the experimental values, whereas in the interval $0.5 \leqslant \xi \leq 1$, it is curve 2 (in which revaporization is not accounted for) which gives the better agreement.

For values of $\xi \simeq 0.5$ both curves give approximately the same results. Applying curve 1 for the interval $0 < \xi \leq 0.5$ and curve 2 for the interval $0.5 \leq \xi \leq 1$, we obtain a theoretical distribution of condensation speeds for all values of the dimensionless coordinate $\xi$ (the dashed curve in Fig. 3). Curves for the variation of the other vapor parameters, $P = P(\xi)$, $\rho = \rho(\xi)$, $T = T(\xi)$, etc., may be obtained in an analogous way.

For the region $0 < \xi \geqslant 0.7$, the experimentally determined condensation speed distribution along the $\xi$ coordinate is independent of the orientation of the condensing surface, confirming thereby the validity of the assumption of a continuous medium. For $\xi \geqslant 0.7$, the experimental relation $w^* = w^*(\xi)$ depends upon whether the condensing surface is placed perpendicular or parallel to the $\xi$ direction; in the former case, $w^*(\xi)$ is retarded in relation to the theoretical value and in the latter case the retardation is much less. The explanation is that in the presence of a "divergent" effect (condensation) the vapor pressure in the chamber decreases depending upon the size of $\xi$. This leads to an increase in the free molecular path length and a gradual degeneration of the gas-dynamic flow into a molecular beam directed along the $\xi$ axis.

Thus the qualitative model presented in [1], and the theoretical analysis of the vaporization, mass transfer, and condensation in a quasiclosed volume carried out in the present paper, enable us to determine the vapor parameters at an arbitrary section of the volume, give good agreement with experiment, and may be applied in engineering calculations.

# APPENDIX

In Eq. (16) we introduced the notation $Q = (1 + 0.2M^2/1.2$. It is obvious that $Q|_{\xi=0} = 1$. Then

$$Q' = \frac{dQ}{d\xi} = \frac{1}{3} MM'; \; Q'|_{\xi=0} = \frac{1}{3} M'|_{\xi=0} = \frac{m_1}{3}; \; \frac{dQ}{dM} = \frac{M}{3}. \tag{A-1}$$

Expanding the exponential factor in Eq. (16) in powers of $b\xi/(1-c\xi$, we have

$$\frac{dM}{d\xi} = aQ\frac{1-Q^3}{M^2-1} + \frac{abQ^4}{M^2-1}\cdot\frac{\xi}{1-c\xi}\times\left[1 - \frac{1}{2}\cdot\frac{b\xi}{1-c\xi} + \frac{1}{6}\left(\frac{b\xi}{1-c\xi}\right)^2 + \dots\right]. \tag{A-2}$$

We obtain $M^2(\xi)$ from Eq. (18):

$$M^2(\xi) = 1 + 2m_1\xi + (m_1^2 + m_2)\xi^2 + \frac{1}{3}(3m_1m_2 + m_3)\xi^3 + \dots \tag{A-3}$$

We substitute Eq. (A-3) into the second term of Eq. (A-2):

$$\frac{dM}{d\xi} = aQ\frac{1-Q^3}{M^2-1} + \frac{abQ^4}{1-c\xi}\times\frac{1 - \frac{1}{2}\frac{b\xi}{1-c\xi} + \frac{1}{6}\left(\frac{b\xi}{1-c\xi}\right)^2 + \dots}{2m_1 + (m_1^2 + m_2)\xi + \frac{1}{3}(3m_1m_2 + m_3)\xi^2 + \dots} \tag{A-4}$$

Then

$$\left.\frac{dM}{d\xi}\right|_{\xi=0} = m_1 = L_1 + \frac{ab}{m_1}, \tag{A-5}$$

$$L_1 = \lim_{\xi\to 0}\left(aQ\frac{1-Q^3}{M^2-1}\right) = a\lim_{M\to 1}\frac{1-Q^3}{M^2-1} = -\frac{a}{2}. \tag{A-6}$$

Substituting Eq. (A-6) into Eq. (A-5), we obtain a quadratic equation in $m_1$:

$$m_1^2 + \frac{a}{2}m_1 - \frac{ab}{2} = 0,$$

whence

$$m_1 = -\frac{a}{4}\left(1 \pm \sqrt{1 + \frac{8b}{a}}\right). \tag{A-7}$$

The sign before the radical in Eq. (A-7) is chosen from the condition $m_1 > 0$, since $M(\xi) > 1$ for $\xi > 0$:

$$\begin{aligned}\frac{d^2M}{d\xi^2} &= aQ'\frac{1-Q^3}{M^2-1} + aQ\frac{-3Q^2(M^2-1)\frac{dQ}{dM} - 2M(1-Q^3)}{(M^2-1)^2}\cdot\frac{dM}{d\xi} \\ &+ ab\frac{4Q^3Q'(1-c\xi) + cQ^4}{1-c\xi}\cdot\frac{1 - \frac{1}{2}\left(\frac{b\xi}{1-c\xi}\right) + \frac{1}{6}\left(\frac{b\xi}{1-c\xi}\right)^2 + \dots}{2m_1 + (m_1^2 + m_2)\xi + \frac{1}{3}(3m_1m_2 + m_3)\xi^2 + \dots} \\ &+ \frac{abQ^4}{1-c\xi}\left\{\left[-\frac{b}{2}\cdot\frac{1}{(1-c\xi)^2} + \dots\right]\left[2m_1 + (m_1^2 + m_2)\xi + \dots\right]\right. \\ &\left.-\left(1 - \frac{1}{2}\frac{b\xi}{1-c\xi}\right)\left[(m_1^2 + m_2) + \dots\right]\right\}\times\{[2m_1 + (m_1^2 + m_2)\xi + \dots]^2\}^{-1},\end{aligned} \tag{A-8}$$

$$\left.\frac{d^2M}{d\xi^2}\right|_{\xi=0} = m_2 = L_1^{(1)} + L_2 + \frac{ab}{2m_1}\left(\frac{4}{3}m_1 + c\right) + ab\frac{-bm_1 - (m_1^2 + m_2)}{4m_1}, \tag{A-9}$$

$$L_1^{(1)} = \lim_{\xi\to 0}\left(aQ'\frac{1-Q^3}{M^2-1}\right) = \frac{am_1}{3}\lim\frac{1-Q^3}{M^2-1} = \frac{am_1}{6}, \tag{A-10}$$

$$\begin{aligned}L_2 &= \lim_{\xi\to 0}\left[-aQ\frac{Q^2(M^2-1)M + 2M(1-Q^3)}{(M^2-1)^2}\frac{dM}{d\xi}\right] \\ &= am_1\lim_{M\to 1}\frac{Q^2(M^2-1) + 2(1-Q^3)}{M^2-1} = \frac{am_1}{6}.\end{aligned} \tag{A-11}$$

Substituting Eqs. (A-10) and (A-11) into Eq. (A-9):

$$m_2 = -\frac{am_1}{6} - \frac{am_1}{6} + \frac{ab}{2m_1}\left(\frac{4}{3}m_1 + c\right) - ab\frac{bm_1 + m_1^2 + m_2}{4m_1^2}$$

or

$$\left(1 + \frac{ab}{4m_1^2}\right)m_2 = -\frac{am_1}{3} + \frac{ab}{2m_1}\left(\frac{5}{6}m_1 + c - \frac{b}{2}\right),$$

whence

$$m_2 = \frac{-\frac{m_1}{3} + \frac{b}{2m_1}\left(c - \frac{b}{2}\right) + \frac{5}{12}b}{1 + \frac{ab}{4m_1^2}}. \tag{A-12}$$

The coefficient $m_3$ is calculated in an analogous way:

$$m_3 = \frac{6m_1^2}{6m_1^2 + ab}\left\{-\frac{a}{3}\left(\frac{13}{9}m_1^2 + m_2\right) + ab\left[\frac{(m_1 + c)\left(\frac{4}{3}m_1 + c\right) + \frac{4}{3}(m_1^2 + m_2)}{2m_1}\right.\right.$$
$$\left.\left. - \frac{2\left(\frac{4}{3}m_1 + c\right)(bm_1 + m_1^2 + m_2) + 2bm_1\left(\frac{b}{3} - c\right) + b(m_1^2 + m_2) - 2m_1m_2}{4m_1^2} + \frac{(m_1^2 + m_2)^2}{4m_1^3}\right]\right\}.$$

In practice, to calculate M = M(ξ) it is entirely satisfactory to use three or four terms of its Maclaurin series expansion.

## NOTATION

| | |
|---|---|
| $T_0$ | is the vaporization temperature; |
| $T_{ch}^{L}$ | is the temperature of the top of the chamber; |
| $T_W$ | is the temperature of the chamber wall; |
| $T_V$ | is the vapor temperature; |
| $T_{crit}$ | is the critical condensation temperature; |
| $T = E_S/R$ | is the characteristic temperature; |
| $E_S$, $E_S'$ | are the vaporization energies, respectively of 1 mole of bulk material and 1 mole of thin film material; |
| R | is the universal gas constant; |
| $\beta$ | is the vapor molecule "adhesion" coefficient; |
| $j_c$, $j_r$ | are the mass fluxes for condensation and revaporization, respectively; |
| G, P, $\rho$ | are the vapor mass flow rate, pressure, and density; |
| u | is the gas-dynamic flow velocity; |
| $G_{crit}$, $P_{crit}$, $\rho_{crit}$, $u_{crit}$ | are the flow rate, pressure, density, and velocity of the vapor at the critical section; |
| $c_p$, $c_V$ | are the vapor specific heat capacities; |
| $\rho_{sat}$ | is the density of saturated vapor; |
| $G_1$, $P_1$, $T_1$, $\rho_1$, $u_1$ | are the flow rate, pressure, temperature, density, and velocity of vapor at the initial gas-dynamic section; |
| $\mu$ | is the molecular weight; |
| $\rho_m$ | is the density of the material; |
| x | is the coordinate measured from the bottom of the chamber; |
| L, d | are the length and diameter of the chamber. |

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